

# QUIZ IN BAYESIAN STATISTICAL LEARNING, AMSI SUMMER SCHOOL 2022

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## Problem 1

Suppose that  $X \sim \mathcal{N}(0, 1)$  and let

$$Y = \frac{\exp(X)}{1 + \exp(X)}.$$

Find the probability density function of  $Y$ .

## Problem 2

(a.)

Complete the square of the expression  $x^2 + 10x - 3$ , i.e. find  $a, h$  and  $k$  such that

$$x^2 + 10x - 3 = a(x - h)^2 + k.$$

(b.)

Suppose now that  $x, b, c \in \mathbb{R}^p$  and that  $A \in \mathbb{R}^{p \times p}$  is a symmetric matrix. Find  $h \in \mathbb{R}^p$  and  $k \in \mathbb{R}$  such that

$$x^\top Ax + x^\top b + c = (x - h)^\top A(x - h) + k.$$

## Problem 3

(a.)

Let  $A_1, A_2, A_3$  and  $A_4$  be dependent events. Show that

$$\Pr(A_1 \cap A_2 \cap A_3 \cap A_4) = \Pr(A_1) \Pr(A_2|A_1) \Pr(A_3|A_1 \cap A_2) \Pr(A_4|A_1 \cap A_2 \cap A_3).$$

(b.)

Suppose now that  $A_1, A_2, A_3, A_4$  are independent events. What is the probability in (a.)?

## Problem 4

Let  $\mathcal{N}(x|\mu_1, \sigma_1^2)$  and  $\mathcal{N}(x|\mu_2, \sigma_2^2)$  denote two normal densities. Show that the product of the two densities is proportional to a normal density. What is the expected value and variance of this new density?