Quiz in Bayesian Statistical Learning, AMSI Summer School 2022

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Problem 1

Suppose that $X \sim \mathcal{N}(0, 1)$ and let

$$Y = \frac{\exp(X)}{1 + \exp(X)}.$$

Find the probability density function of Y.

Problem 2

(a.)

Complete the square of the expression $x^2 + 10x - 3$, i.e. find a, h and k such that

$$x^2 + 10x - 3 = a(x - h)^2 + k.$$

(b.)

Suppose now that $x, b, c \in \mathbb{R}^p$ and that $A \in \mathbb{R}^{p \times p}$ is a symmetric matrix. Find $h \in \mathbb{R}^p$ and $k \in \mathbb{R}$ such that

$$x^\top Ax + x^\top b + c = (x - h)^\top A(x - h) + k.$$

Problem 3

(a.)

Let A_1, A_2, A_3 and A_4 be dependent events. Show that

$$\Pr(A_1 \cap A_2 \cap A_3 \cap A_4) = \Pr(A_1)\Pr(A_2|A_1)\Pr(A_3|A_1 \cap A_2)\Pr(A_4|A_1 \cap A_2 \cap A_3).$$

(b.)

Suppose now that A_1, A_2, A_3, A_4 are independent events. What is the probability in $(a.)$?

Problem 4

Let $\mathcal{N}(x|\mu_1, \sigma_1^2)$ and $\mathcal{N}(x|\mu_2, \sigma_2^2)$ denote two normal densities. Show that the product of the two densities is proportional to a normal density. What is the expected value and variance of this new density?

For quiz solutions, please click here