A Modern Introduction to Extreme Value Theory

AMSI Summer School, 2022

Quiz

Exercise 1

Consider a real-valued random variable X with distribution function F_X and probability density function f_X. Let $X_1, \ldots, X_n$ denote n independent and identically distributed replicates of X.

1. Define the random variable $Z = \min(X_1, \ldots, X_N)$. What are its distribution function (F_Z) and probability density function (f_Z)?
2. What is the joint distribution function and probability density function of (Y, Z)?
3. Assume that X is in fact uniformly distributed on $[0, 1]$. According to which common distributions are Y and Z distributed? Write down the joint probability density function of (Y, X).

Exercise 2

Approximately 40% of the Netherlands is below sea level. Much of it has to be protected against the sea by dykes. These dykes have to withstand storm surges that drive the seawater level up along the coast. The Dutch government, balancing considerations of cost and safety, has determined that the dykes should be sufficiently high that the probability of a flood (which occurs when the seawater level exceeds the top of the dyke) in a given year is 10^{-4}. The question is then: how high should the dykes be built to meet this requirement?

1. Using `R`, simulate weekly observations of the sea level (in metres) at the town of Delfzijl between the January 1st 2002 and January 1st 2022, with each observation being drawn from the Exponential distribution with rate 2. For reproducibility use `set.seed(123)`.
2. Assuming that the Exponential distribution is the true underlying distribution ($F_X(x)$), and using the simulated data, calculate the height that the dyke should be built at.
3. Repeat the previous question assuming that the Gamma distribution is the true underlying distribution.
4. Display a histogram of the observed yearly maxima of the sea level and add the dyke heights calculate in the previous questions.

Find solutions here