Practical Asymptotics

AMSI Summer School 2021

Pre-course quiz

The quiz below is partly on assumed knowledge, and partly some exercise on the use of MATLAB. Note that during the course example codes will be provided for you to adapt (so don't worry if you haven't seen some of the material below before).

---

1. **Power series solution of an ODE**

   Consider the following initial value problem

$$y'' + (1 + x^2)y = 0, \quad \text{with } y(0) = 1, y'(0) = 1.$$

a) Find a representation of the general solution to this ODE as a power series about $x = 0$. Write out the solution up to and including the x^7 term (which gives four terms in each of the two linearly independent solutions).

b) Check this solution in MATLAB using `dsolve` with the option `ExpansionPoint`. Note that this is a new feature in recent (from 2020a) versions of MATLAB. If you're unfamiliar with using MATLAB's Symbolic Toolbox for computer algebra, a good place to start would be the documentation on `dsolve`.

c) Apply the initial conditions that $y(0) = 1$ and $y'(0) = 1$. Compare your series solution to a numerical solution calculated with MATLAB's `ode45` routine (maybe plot the two on top of each other). Again, it may help to check out MATLAB's documentation to get familiar with the numerical solution of ODEs.

---

2. **Numerical solution of a boundary value problem**

   Use MATLAB's `bvp4c` routine to find a numerical solution to the boundary value problem for $y(x)$

$$0.003y'' - \sin(x)y = 1, \quad -4\pi < x < 4\pi,$$

   subject to the boundary conditions $y(-4\pi) = y(4\pi) = 0$. The syntax for this is similar to `ode45`. As above, check out MATLAB's documentation for full details.

   Having found your solution, make a plot.

---

3. **Integrals in the complex plane**

   [Note that we'll revise the necessary complex calculus at the relevant point in the course.]

   Consider the following integral

$$\oint_C \frac{1}{z^2(z-2)} \mathrm{d}z$$

   where the contour C is the unit circle in the complex plane (traversed anti-clockwise).

a) Use residue theorem to evaluate the integral.

b) Investigate using the `poles` function in MATLAB and use it to check some of the detail in your working to part (a).

c) Read MATLAB's documentation on "Complex Line Integrals" and adapt the examples to evaluate the above integral numerically (perhaps deform the original contour first).