1. Questions

AMSI SS2020: FINITE ELEMENT METHOD – PRE-QUIZ

Instructions. The following questions assume you have some background in analysis and linear algebra. We expect the quiz to take less than one hour.

(1) Let V be a vector space. Define basis and dimension of V. If W is the vector space of polynomials of degree at most 2 on $\mathbb{R}$, write down its basis and its dimension.

(2) Consider the vector space of polynomials of degree at most 3 on $\mathbb{R}$. Which ones are not the basis of this polynomial space?

A. $\{1, x, x^2, x^3\}$

B. $\{0, 1, x, x^2, x^3\}$

C. $\{1 + x, 1 - x, x^2, x^3\}$

D. $\{1 + x, x, 1 - x^2, 1 + x^3\}$

(3) What is the largest k for the function $f(x) = |x|$ so that $f \in C^k(\mathbb{R})$?

A. 1

B. 0

C. 2

D. 3

(4) Let $\Omega = [0, 1] \times [0, 1]$ and $u : \Omega \to \mathbb{R}$ be defined as

$$u(x, y) = x^2 + y^2.$$

Compute the norms $\|u\|_{C^1(\Omega)}$ and $\|u\|_{L^2(\Omega)}$.

(5) Which of the following statements are true for an open bounded domain Ω with a smooth boundary? Here $\bar{\Omega} = \Omega \cup \partial\Omega$ and $\partial\Omega$ is the boundary of Ω.

A. $L^2(\Omega) \subset C^1(\Omega)$.

B. $C^2(\Omega) \subset C^1(\Omega)$

C. $C^0(\bar{\Omega}) \subset L^2(\bar{\Omega})$

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D. $C^2(\Omega) \subset C^3(\Omega)$

(6) Let $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = |x|$. Find the weak derivative of f.

(7) The sine function has the power series definition

$$\sin(x) = \sum_{k=0}^{\infty} (-1)^k \frac{x^{2k+1}}{(2k+1)!}.$$

Write a function `SineTaylor.m` that has input n and x and output the relative error in the partial sums

$$S_n(x) = \sum_{k=0}^{n} (-1)^k \frac{x^{2k+1}}{(2k+1)!}.$$

The relative error R_n is then defined as

$$R_n = \frac{|\sin(x) - S_n(x)|}{|\sin(x)|}.$$

Think of an efficient way of computing S_n. You can start your function file in the following way:

```
function relErr = SineTaylor(x,n)
% SineTaylor(x,n) evaluates the relative error between sin(x)
% and its n-term MacLaurin series at x
% Usage: relErr=SineTaylor(x,n)
%inputs: x point where we want to compute the error
%        n number of terms in MacLaurin series
%output: relErr the relative error in approximation
```

Plot the errors for $n = 2, 4, 6, 8, 10, 12, 14, 16, 18, 20$ using $x = 4$ using the logarithmic scale. What happens when n is large?

2. Answers

(1) Let B be a subset of linearly independent elements in V and $V = \text{Span}(B)$, where $\text{Span}(B)$ is the set of all possible linear combinations of elements in B. Then the set B is a basis of V. If B has a finite number of elements, the number of elements in B is the dimension of V. A basis of W is $\{1, x, x^2\}$. Its dimension is 3.

(2) B, D

(3) B

(4) 6 and $\sqrt{\frac{28}{45}}$

(5) B,C

(6) The weak derivative h of the given function f is

$$h(x) = \begin{cases} -1 & \text{if } \quad x < 0 \\ 1 & \text{if } \quad x > 0. \end{cases}$$

(7) Include the following lines of code in the function defintion.

```
r1=[0:1:n];
r2=(-1).^(r1);
Sn=sum(r2.*(x*(x.^(2*r1))./factorial(2*r1+1)));
relErr=abs(sin(x)-Sn)/abs(sin(x));
```

You can use the following piece of MATLAB code to plot the result.

```
idx =2:2:20;
for j=1:length(idx)
RE(j) =SineTaylor(4,idx(j));
end
loglog(idx,RE,'-*','markersize',20);
```