Pre-enrollment quiz

If necessary, take time to refresh your memory of the concepts in the questions below (it is okay to ask others for assistance).

After refreshing your memory, you should be able to complete almost all of these problems from scratch without reference to any other resources.

1. Give the first three non-zero terms of the Taylor expansion of $-2x + e^{-2(1-x)}$.
2. For $|x| < 1$, compute $S(x) = 1 + x + x^2 + x^3 + \cdots$ in closed form.
3. Use integration by substitution to find $\int_0^{2\pi} \cos(2x)\sin(2x)\,dx$
4. Taking $i = \sqrt{-1}$, find the integral $\int_0^{2\pi} 2e^{3i\theta}\,d\theta$ by using $e^{ix} = \cos(x) + i\sin(x)$ before integrating.
5. Repeat the integral above by direct integration treating i as a constant. Only use $e^{ix} = \cos(x)+i\sin(x)$ in the final step of evaluating the numerical value of the integral.
6. Find the points in the complex plane given by $e^{i\theta}$ for $0 \le \theta \le 2\pi$.
7. If a weighted coin has probability 0.4 of heads and 0.6 of tails, what is the expected number of heads after 10 flips?
8. For the 10 flips of the coin above, what is the probability the first flip is heads and the last is tails?
9. For the 10 flips of the coin above, what is the probability of no heads? What is the probability of at least one heads?
10. For the 10 flips of the coin above, what is the probability that all flips are the same (hint, break this into the cases of all heads or all tails)?
11. Let $M = \begin{pmatrix} 2 & 1 \\ 3 & 4 \end{pmatrix}$. Compute $\det M$.
12. For the same matrix M, find its eigenvalues and eigenvectors.
13. For the eigenvalues/eigenvectors above, verify by direct calculation that $M\vec{v} = \lambda\vec{v}$ for each pair $\vec{v}$ and λ
14. If $\vec{v}$ and $\vec{w}$ are eigenvectors with eigenvalues λ_v and λ_w, found above, write down a formula for $M^N(2\vec{v} + 5\vec{w})$ for any non-negative integer N
15. Solve the first-order linear ODE $\dfrac{dy}{dt} = -2y$, given $y(0) = 5$.
16. A population size $P(t)$ solves the equation $\dfrac{dP}{dt} = 0.3\,P(t)$. Express $P(t)$ in terms of $P(0)$.

17. Assume there are 8 students in a classroom. Come up with a question you could ask about those 8 students whose answer is $\binom{8}{3}$. Compute the value.

18. Expand $(2+x)^4$ completely.

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